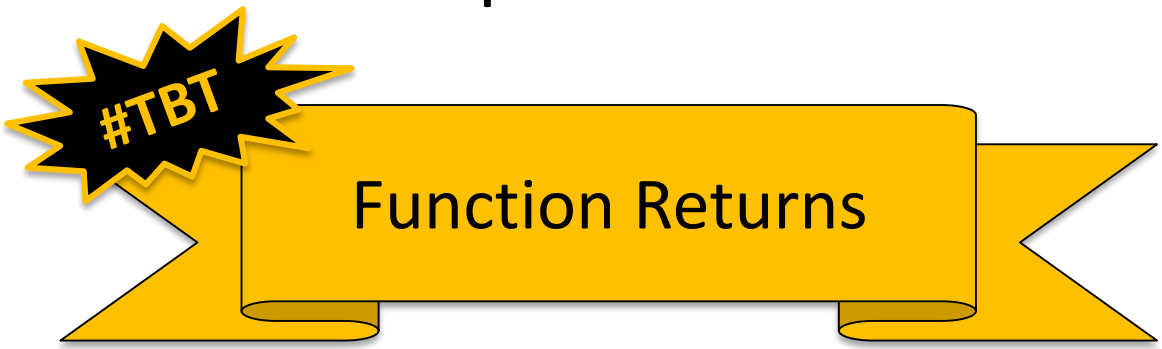
CMSC201
Computer Science I for Majors

Lecture 16 – Recursion

Last Class We Covered

- What makes "good code" good
  - Readability
  - Adaptability
  - Commenting guidelines
- Incremental development



Any Questions from Last Time?

Today's Objectives

- To introduce recursion
- To better understand the concept of "stacks"
- To begin to learn how to "think recursively"
  - To look at examples of recursive code
  - Summation, factorial, etc.

Introduction to Recursion

What is Recursion?

- In computer science, ***recursion*** is a way of thinking about and solving problems
- It's actually one of the central ideas of CS

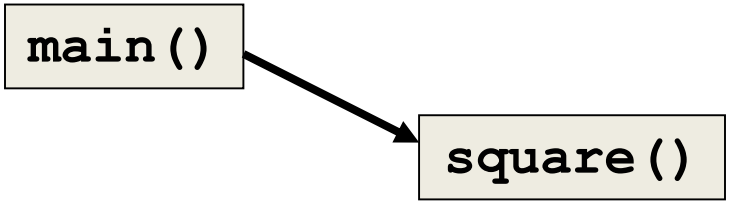
- In recursion, the solution depends on solutions to smaller instances of the same problem

Recursive Solutions

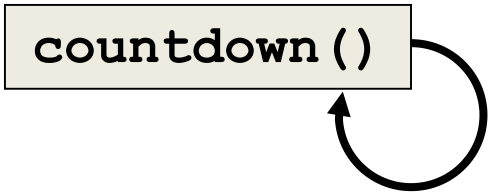
- When creating a recursive solution, there are a few things we want to keep in mind:
  1. We need to break the problem into smaller pieces of itself
  2. We need to define a “base case” to stop at
  3. The smaller problems we break down into need to eventually reach the base case

Normal vs Recursive Functions

- So far, we've had functions call other functions
  - For example, **`main()`** calls the **`square()`** function



- A recursive function, however, calls itself



Why Would We Use Recursion?

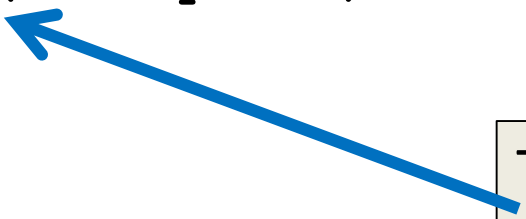
- In computer science, some problems are more easily solved by using recursive methods
- For example:
  - Traversing through a directory or file system
  - Traversing through a tree of search results
  - Some sorting algorithms recursively sort data
- For today, we will focus on the basic structure of using recursive methods

Toy Example of Recursion

```
def countdown(intInput):
    print(intInput)
    if (intInput > 2):
        countdown(intInput-1)

def main():
    countdown(50)

main()
```



This is where the recursion occurs.

You can see that the **`countdown()`** function calls itself.

What does this program do?

This program prints the numbers from 50 down to 2.

Visualizing Recursion

- To understand how recursion works, it helps to visualize what's going on
- Python uses a ***stack*** to keep track of function calls
- A stack is an important computer science concept

Stacks

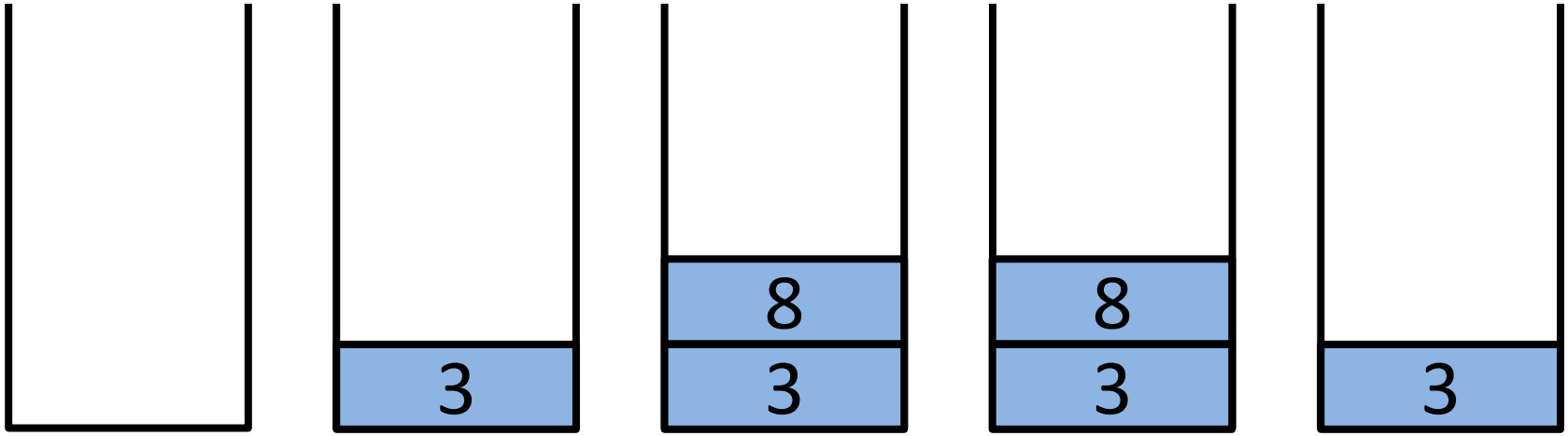
Stacks

- A stack is like a bunch of lunch trays in a cafeteria
- It has only two operations:
  - Push
    - You can push something onto the top of the stack
  - Pop
    - You can pop something off the top of the stack

- Let's see an example stack in action

Stack Example

- In the animation below, we “push 3”, then “push 8”, and then pop twice



Stack Details

- In computer science, a stack is a ***last in, first out*** (LIFO) data structure

- It can store any type of data, but has only two operations: push and pop
- Push adds to the top of the stack, hiding anything else on the stack
- Pop removes the top element from the stack

Stack Details

- The nature of the pop and push operations also means that stack elements have a natural order

- Elements are removed from the stack in the reverse order to the order of their addition
  - The lower elements are those that have been in the stack the longest

Stack Exercise

- In your notebooks, trace the following commands, to the stack's final appearance

1. Push "D"
2. Push "O"
3. Push "D"
4. Pop
5. Push "G"
6. Push "M"
7. Push "K"
8. Pop
9. Push "T"
10. Pop
11. Pop
12. Push "G"
13. Push "E"
14. Push "F"
15. Pop
16. Push "H"
17. Pop
18. Push "S"

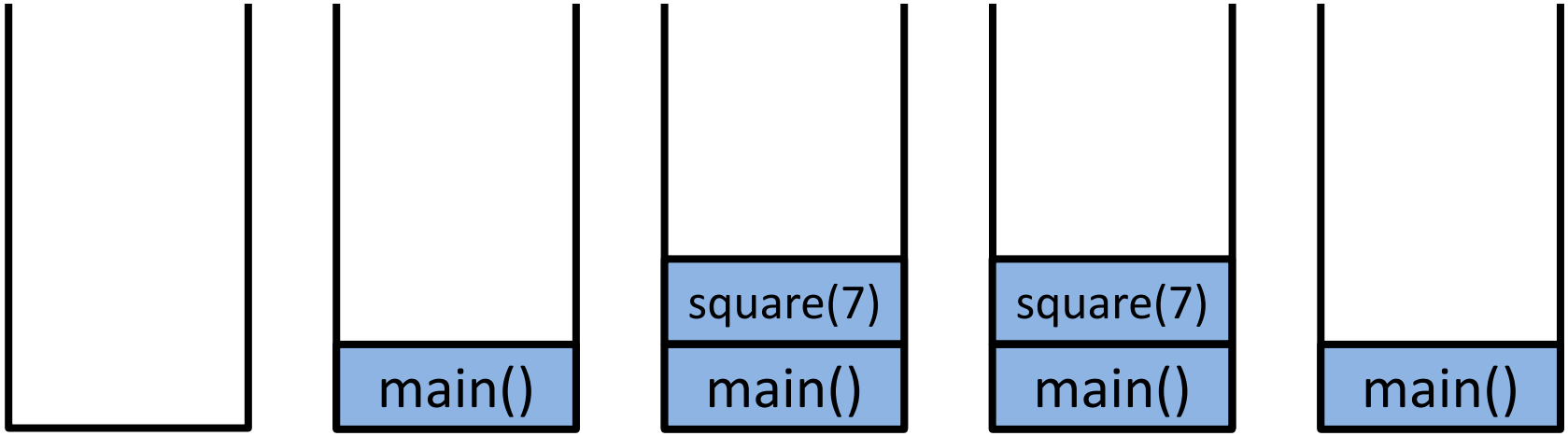
Stacks and Functions

- When you run your program, the computer creates a stack for you

- Each time you call a function, the function is pushed onto the top of the stack
- When the function returns or exits, the function is popped off the stack

Stack Example

- Run



Stacks and Recursion

- If a function calls itself recursively, you push another call to the function onto the stack

- We now have a simple way to visualize how recursion really works

Toy Example of Recursion

```
def countdown(intInput):
    print(intInput)
    if (intInput > 2):
        countdown(intInput-1)

def main():
    countdown(50)

main()
```

Here's the code again.

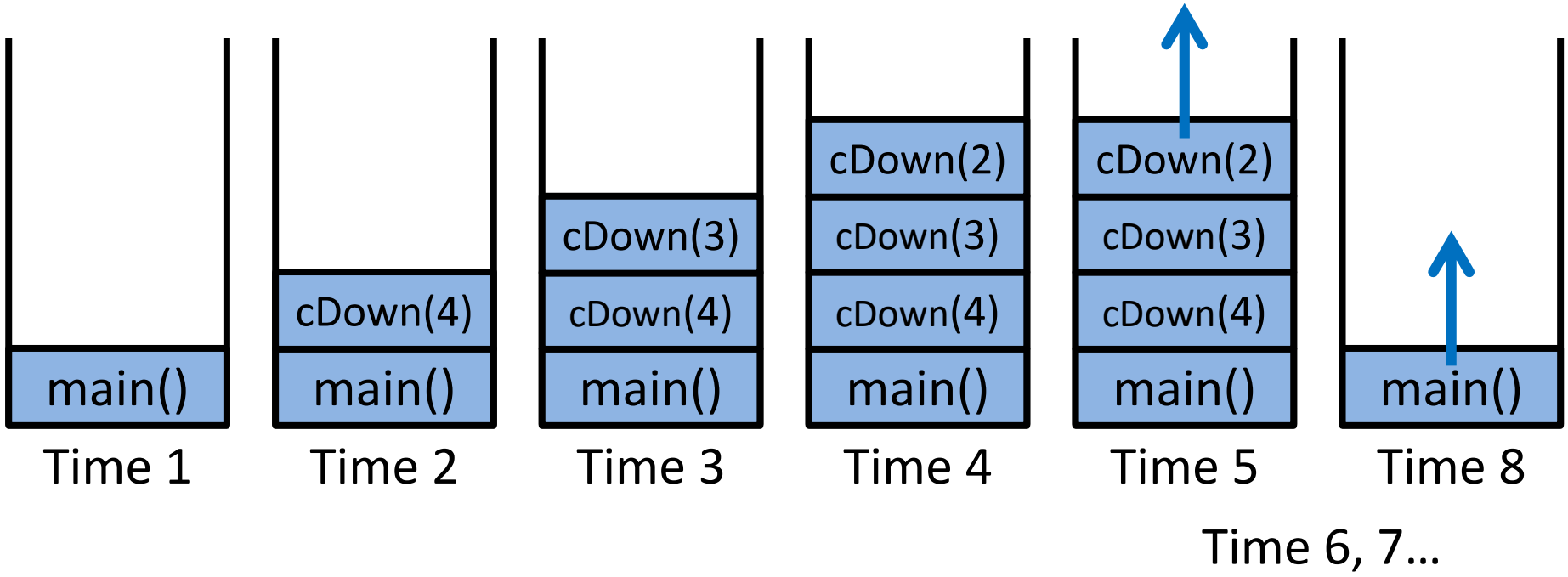
Now, that we understand stacks, we can visualize the recursion.

We'll call the function with a value of just 4 for the trace.

We'll also shorten it to **`cDown()`**.

Stack and Recursion in Action

- Skipping time step 0, start by pushing main()…



Defining Recursion

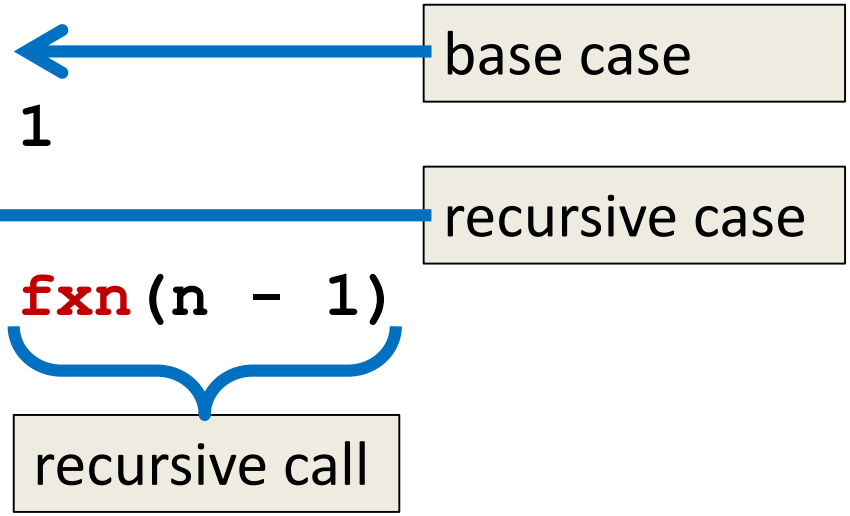
"Cases" in Recursion

- A recursive function must have two things:

- At least one base case
  - When a result is returned (or the function ends)
  - "When to stop"
- At least one recursive case
  - When the function is called again with new inputs
  - "When to go (again)"

Terminology

```
def fxn(n):
    if n == 1:
        return 1
    else:
        return fxn(n - 1)
```



- Notice that the recursive call is passing in simpler input, approaching the base case

Recursion Example

```
def summ(n):
    if n == 1:
        return 1
    else:
        return n + summ (n - 1)
```

- What is **summ (1)** ?
- What is **summ (2)** ?
- What is **summ (100)** ?
  - We at least know that it's **100 + summ(99)**

Recursion Example

```
def summ(n):
    if n == 1:
        return 1
    else:
        return n + summ (n - 1)
```

```
summ(3)
    3 + summ(2)
            2 + summ(1)
                    1
    3 +     2 +     1   =   6
```

Factorials

- $4! = 4 \times 3 \times 2 \times 1 = 24$

- Does anyone know the value of 9! ?
- 362,880

- Does anyone know the value of 10! ?
- How did you know?

Factorial

- $9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
- $10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
- $10! = 10 \times 9!$

- $n! = n \times (n - 1)!$

- That's a recursive definition!
  - The answer to a problem can be defined as a smaller piece of the original problem

Factorial

```
def fact(n):
    return n * fact(n - 1)

fact(3)
3 * fact(2)
    2 * fact(1)
        1 * fact(0)
            0 * fact(-1)
...
```

What happened? What went wrong?

Factorial (Fixed)

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n - 1)

fact(3)
3 * fact(2)
    2 * fact(1)
        1 * fact(0)
            1
```

Recursion Practice

Thinking Recursively

- Anything we can do with a **`while`** loop can also be accomplished through recursion
- Let's get some practice by transforming basic loops into a recursive function

- To keep in mind:
  - What is the base case? The recursive case?
  - Are we returning values, and if so, how?

Non-Recursive `sumList()`

- Sum the contents of a list together

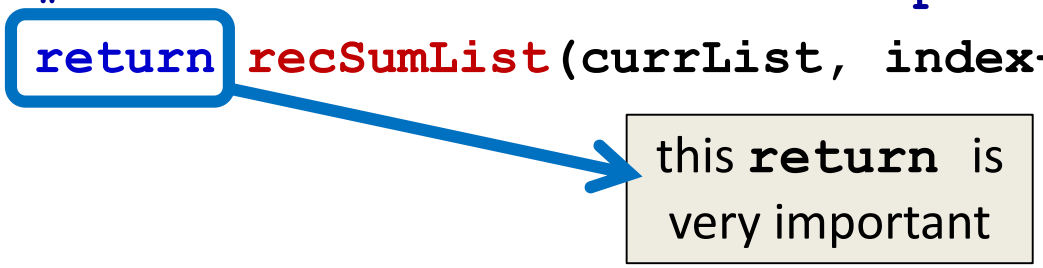
```
def sumList(numList):
    total = 0
    for i in range(len(numList)):
        total = total + numList[i]
    return total
```

- Transform this into a recursive function

Recursive `sumList()`

- Recursively sum the contents of a list together

```
def recSumList(currList, index, total):
    # BASE CASE: reached the end of the list
    if index == len(currList):
        return total
    else:
        total += currList[index]
        # RECURSIVE CALL: call with updated index
        return recSumList(currList, index+1, total)
```



Recursive `sumList()`

- Recursively sum the contents of a list together

```
def recSumList(currList, index):
    # BASE CASE: reached the end of the list
    if index == len(currList):
        return 0
    else:
        # RECURSIVE CALL: add this element to rest of list
        print("Adding", currList[index], "to total")
        return currList[index] + \
            recSumList( currList, index+1 )
```

Recursive Thinking

- Sometimes, creating a recursive function requires us to think about the problem differently

- What kind of base case do we need for summing a list together? How do we know we're "done"?
  - Instead of approaching the problem as before, you could think of it instead as adding the first element to the sum of the rest of the list

Recursive Summing

```
myList = [3,  5,  8,  7,  2,  6,  1]
          3 + [5, 8, 7, 2, 6, 1]
              5 + [8, 7, 2, 6, 1]
                  8 + [7, 2, 6, 1]
                      etc...
```

- What is the base case here?
- How does the recursive case work?

Recursive `sumList()`

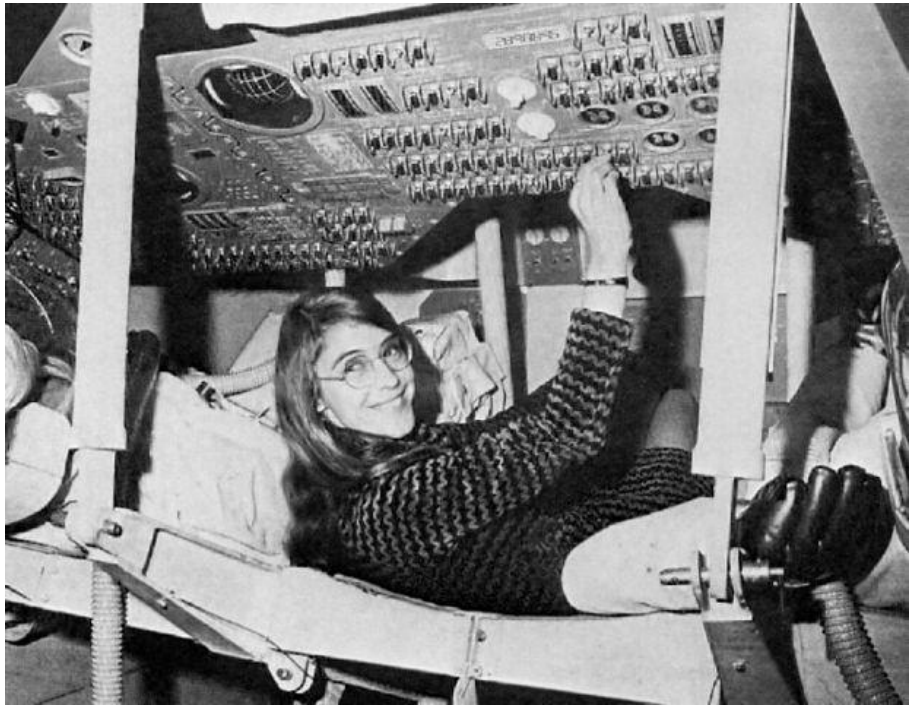
- Recursively sum the contents of a list together

```
def recSumList(currList):
    # BASE CASE: no elements left (empty list)
    if len(currList) == 0:
        return 0
    else:
        # RECURSIVE CALL: add first element to rest of list
        print("Adding", currList[0], "to", currList[1:] )
        return currList[0] + recSumList( currList[1:] )
```

again, this **return** is very important

Daily CS History

- Margaret Hamilton
  - Who is she?
    - The original Hamilton
  - We'll cover her next time



Announcements

- Project 2 is out on Blackboard now
  - Design is due by Friday (Apr 12th) at 11:59:59 PM
  - Project is due by Friday (Apr 19th) at 11:59:59 PM
- Significantly more difficult than Project 1
  - Probably at least at 10 hour project (closer to 15)
- Second midterm exam is **April 17th and 18th**

Image Sources

- Pancake drizzle:
  - http://www.topwithcinnamon.com/2013/01/2-ingredient-healthy-pancakes-gluten-free-dairy-free.html
- Margaret Hamilton
  - https://en.wikipedia.org/wiki/File:Margaret_Hamilton_in_action.jpg